8-3 Study Guide and Intervention

Graphing Reciprocal Functions

Vertical and Horizontal Asymptotes

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<th>Parent Function of Reciprocal Functions</th>
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<td>Parent Function</td>
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Example: Identify the asymptotes, domain, and range of the function \( f(x) = \frac{3}{x + 2} \).

Identify \( x \) values for which \( f(x) \) is undefined.
\( x + 2 = 0 \), so \( x = -2 \). \( f(x) \) is not defined when \( x = -2 \), so there is an asymptote at \( x = -2 \).

From \( x = -2 \), as \( x \)-values decrease, \( f(x) \) approaches 0. As \( x \)-values increase, \( f(x) \) approaches 0. So there is an asymptote at \( f(x) = 0 \).

The domain is all real numbers not equal to \(-2\), and the range is all real numbers not equal to 0.

Exercises

Identify the asymptotes, domain, and range of each function.

1. \( f(x) = \frac{1}{x} \)
2. \( f(x) = \frac{-3}{x - 1} \)
3. \( f(x) = \frac{4}{x + 1} + 2 \)
8-3 Study Guide and Intervention (continued)
Graphing Reciprocal Functions

Transformations of Reciprocal Functions

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<th>Equation Form</th>
<th>( f(x) = \frac{a}{x - h} + k )</th>
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<td>Horizontal Translation</td>
<td>The <em>vertical</em> asymptote moves to ( x = h ).</td>
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<td>Vertical Translation</td>
<td>The <em>horizontal</em> asymptote moves to ( y = k ).</td>
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<td>Reflection</td>
<td>The graph is reflected across the ( x )-axis when ( a &lt; 0 ).</td>
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<tr>
<td>Compression and Expansion</td>
<td>The graph is compressed vertically when (</td>
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Example: Graph \( f(x) = \frac{-1}{x + 1} - 3 \). State the domain and range.

\( a < 0 \): The graph is reflected over the \( x \)-axis.

\( 0 < |a| < 1 \): The graph is compressed vertically.

\( h = -1 \): The *vertical* asymptote is at \( x = -1 \).

\( k = -3 \): The *horizontal* asymptote is at \( f(x) = -3 \).

\( D = \{x \mid x \neq -1\}; R = \{f(x) \mid f(x) \neq -3\} \)

Exercises

Graph each function. State the domain and range.

1. \( f(x) = \frac{1}{x} + 1 \)
2. \( f(x) = \frac{-2}{x - 2} \)
3. \( f(x) = \frac{-1}{x - 3} \)
4. \( f(x) = \frac{1}{x + 5} + 3 \)
5. \( f(x) = \frac{-2}{x - 1} + 2 \)
6. \( f(x) = \frac{1}{x - 3} + 4 \)
8-3 Graphing Calculator Activity
Finding Asymptotes for Reciprocal Functions

Reciprocal equations can be evaluated on a graphing calculator or on a TI-Nspire.

Example: Estimate the asymptotes of \( y = \frac{5}{x+1} + 2 \) by graphing the function on a graphing calculator.

Enter the equation into the calculator.

Keystrokes: \( \text{Y=} \ 5 \div (x,\theta,n) + 1 \) \( + \ 2 \)

Next graph the equation.

Keystrokes: \( \text{GRAPH} \)

Now use the trace function to find the coordinates of the asymptotes. Use the arrow keys to move the cursor across to estimate the asymptotes and read the coordinates at the bottom of the screen.

The asymptotes are \( x = -1 \) and \( y = 2 \).

Exercises

Estimate the asymptotes for these reciprocal equations by graphing the function on a graphing calculator.

1. \( f(x) = \frac{-2}{x-3} - 1 \) 
2. \( f(x) = \frac{-9}{x+8} - 4 \) 
3. \( f(x) = \frac{-5}{x-10} - 2 \) 
4. \( f(x) = \frac{5}{x+7} - 8 \) 
5. \( f(x) = \frac{-3}{x-2} + 6 \) 
6. \( f(x) = \frac{4}{x-7} - 3 \) 
7. \( f(x) = \frac{-2}{x+3} - 4 \) 
8. \( f(x) = \frac{9}{x-3} - 1 \) 
9. \( f(x) = \frac{88}{x+12} - 2 \)