8-4 Study Guide and Intervention

Graphing Rational Functions

**Vertical and Horizontal Asymptotes**

<table>
<thead>
<tr>
<th>Rational Function</th>
<th>A function with an equation of the form ( f(x) = \frac{p(x)}{q(x)} ), where ( p(x) ) and ( q(x) ) are polynomial expressions and ( q(x) \neq 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain</td>
<td>The domain of a rational function is limited to values for which the function is defined.</td>
</tr>
<tr>
<td>Vertical Asymptote</td>
<td>An asymptote is a line that the graph of a function approaches. If the simplified form of the related rational expression is undefined for ( x = a ), then ( x = a ) is a vertical asymptote.</td>
</tr>
<tr>
<td>Horizontal Asymptote</td>
<td>Often a horizontal asymptote occurs in the graph of a rational function where a value is excluded from the range.</td>
</tr>
</tbody>
</table>

**Example:** Graph \( f(x) = \frac{x^2 + x - 6}{x + 1} \).

\[
\frac{x^2 + x - 6}{x + 1} = \frac{(x + 3)(x - 2)}{x + 1}
\]

Therefore the graph of \( f(x) \) has zeroes at \( x = -3 \) and \( x = 2 \) and a vertical asymptote at \( x = -1 \). Because the degree of \( x^2 + x - 6 \) is greater than \( x + 1 \), there is no horizontal asymptote. Make a table of values. Plot the points and draw the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-5)</th>
<th>(-4)</th>
<th>(-3)</th>
<th>(-2)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>(-3.5)</td>
<td>(-2)</td>
<td>(0)</td>
<td>(4)</td>
<td>(-6)</td>
<td>(-2)</td>
<td>(0)</td>
<td>(1.5)</td>
<td>(2.8)</td>
</tr>
</tbody>
</table>

**Exercises**

Graph each function.

1. \( f(x) = \frac{4}{x^2 + 3x - 10} \)

2. \( f(x) = \frac{x^2 - 2x + 1}{x^2 + 2x + 1} \)

3. \( f(x) = \frac{2x + 9}{2x^2 - x - 3} \)
Graphing Rational Functions

Oblique Asymptotes and Point Discontinuity

An oblique asymptote is an asymptote that is neither horizontal nor vertical. In some cases, graphs of rational functions may have point discontinuity, which looks like a hole in the graph.

That is because the function is undefined at that point.

**Oblique Asymptotes**

If \( f(x) = \frac{a(x)}{b(x)} \), \( a(x) \) and \( b(x) \) are polynomial functions with no common factors other than 1 and \( b(x) \neq 0 \), then \( f(x) \) has an oblique asymptote if the degree of \( a(x) \) minus the degree of \( b(x) \) equals 1.

**Point Discontinuity**

If \( f(x) = \frac{a(x)}{b(x)} \), \( b(x) \neq 0 \), and \( x - c \) is a factor of both \( a(x) \) and \( b(x) \), then there is a point discontinuity at \( x = c \).

Example: Graph \( f(x) = \frac{x - 1}{x^2 + 2x - 3} \).

\[
\frac{x - 1}{x^2 + 2x - 3} = \frac{x - 1}{(x - 1)(x + 3)} = \frac{1}{x + 3}
\]

Therefore the graph of \( f(x) \) has an asymptote at \( x = -3 \) and a point discontinuity at \( x = 1 \).

Make a table of values. Plot the points and draw the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2.5</th>
<th>-2</th>
<th>-1</th>
<th>-3.5</th>
<th>-4</th>
<th>-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>2</td>
<td>1</td>
<td>0.5</td>
<td>-2</td>
<td>-1</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

**Exercises**

Graph each function.

1. \( f(x) = \frac{x^2 + 5x + 4}{x + 3} \)

2. \( f(x) = \frac{x^2 - x - 6}{x - 3} \)

3. \( f(x) = \frac{x^2 - 6x + 8}{x^2 - x - 2} \)
8-4 Skills Practice
Graphing Rational Functions

Graph each function.

1. \( f(x) = \frac{-3}{x} \)
   ![Graph of \( f(x) = \frac{-3}{x} \)]

2. \( f(x) = \frac{10}{x} \)
   ![Graph of \( f(x) = \frac{10}{x} \)]

3. \( f(x) = \frac{-4}{x} \)
   ![Graph of \( f(x) = \frac{-4}{x} \)]

4. \( f(x) = \frac{2}{x - 1} \)
   ![Graph of \( f(x) = \frac{2}{x - 1} \)]

5. \( f(x) = \frac{x}{x + 2} \)
   ![Graph of \( f(x) = \frac{x}{x + 2} \)]

6. \( f(x) = \frac{x^2 - 4}{x - 2} \)
   ![Graph of \( f(x) = \frac{x^2 - 4}{x - 2} \)]

7. \( f(x) = \frac{x^2 + x - 12}{x - 3} \)
   ![Graph of \( f(x) = \frac{x^2 + x - 12}{x - 3} \)]

8. \( f(x) = \frac{x - 1}{x^2 - 4x + 3} \)
   ![Graph of \( f(x) = \frac{x - 1}{x^2 - 4x + 3} \)]

9. \( f(x) = \frac{3}{x^2 - 2x - 8} \)
   ![Graph of \( f(x) = \frac{3}{x^2 - 2x - 8} \)]

10. \( f(x) = \frac{x^3}{2x + 2} \)
    ![Graph of \( f(x) = \frac{x^3}{2x + 2} \)]

11. \( f(x) = \frac{2x^3 + 4x^2 - 10x - 12}{2x^2 + 8x + 6} \)
    ![Graph of \( f(x) = \frac{2x^3 + 4x^2 - 10x - 12}{2x^2 + 8x + 6} \)]

12. \( f(x) = \frac{(x + 1)^2}{2x - 1} \)
    ![Graph of \( f(x) = \frac{(x + 1)^2}{2x - 1} \)]