11.7 Events Involving And; Conditional Probability

You are considering a job offer in South Florida. You were thrilled by images of Miami on MTV's The Real World. The job offer is just what you wanted and you are excited about living in the midst of Miami's tropical diversity. However, there is just one thing: the risk of hurricanes. You expect to stay in Miami ten years and buy a home. What is the probability that South Florida will be hit by a hurricane at least once in the next ten years?

In this section, we look at the probability that an event occurs at least once by expanding our discussion of probability to events involving and.

And Probabilities with Independent Events
Consider tossing a fair coin two times in succession. The outcome of the first toss, heads or tails, does not affect what happens when you toss the coin a second time. For example, the occurrence of tails on the first toss does not make tails more likely or less likely to occur on the second toss. The repeated toss of a coin produces independent events because the outcome of one toss does not affect the outcome of others.

INDEPENDENT EVENTS
Two events are independent events if the occurrence of either of them has no effect on the probability of the other.

When a fair coin is tossed two times in succession, the set of equally likely outcomes is

\{heads heads, heads tails, tails heads, tails tails\}.

We can use this set to find the probability of getting heads on the first toss and heads on the second toss:

\[ P(\text{heads and heads}) = \frac{\text{number of ways two heads can occur}}{\text{total number of possible outcomes}} = \frac{1}{4}. \]

We can also determine the probability of two heads, \(\frac{1}{4}\), without having to list all the equally likely outcomes. The probability of heads on the first toss is \(\frac{1}{2}\). The probability of heads on the second toss is also \(\frac{1}{2}\). The product of these probabilities, \(\frac{1}{2} \cdot \frac{1}{2}\), results in the probability of two heads, namely \(\frac{1}{4}\). Thus,

\[ P(\text{heads and heads}) = P(\text{heads}) \cdot P(\text{heads}). \]

In general, if two events are independent, we can calculate the probability of the first occurring and the second occurring by multiplying their probabilities.
And PROBABILITIES WITH INDEPENDENT EVENTS

If \( A \) and \( B \) are independent events, then
\[
P(A \text{ and } B) = P(A) \cdot P(B).
\]

**EXAMPLE 1** Independent Events on a Roulette Wheel

Figure 11.11 shows a U.S. roulette wheel that has 38 numbered slots (1 through 36, 0, and 00). Of the 38 compartments, 18 are black, 18 are red, and 2 are green. A play has the dealer spin the wheel and a small ball in opposite directions. As the ball slows to a stop, it can land with equal probability on any one of the 38 numbered slots. Find the probability of red occurring on two consecutive plays.

**Solution** The wheel has 38 equally likely outcomes and 18 are red. Thus, the probability of red occurring on a play is \( \frac{18}{38} \), or \( \frac{9}{19} \). The result that occurs on each play is independent of all previous results. Thus,
\[
P(\text{red and red}) = P(\text{red}) \cdot P(\text{red}) = \frac{9}{19} \cdot \frac{9}{19} = \frac{81}{361} \approx 0.224.
\]

The probability of red occurring on two consecutive plays is \( \frac{81}{361} \).

Some roulette players incorrectly believe that if red occurs on two consecutive plays, then another color is "due." Because the events are independent, the outcomes of previous spins have no effect on any other spins.

**EXERCISE 1** Find the probability of green occurring on two consecutive plays on a roulette wheel.

The and rule for independent events can be extended to cover three or more independent events. Thus, if \( A, B, \) and \( C \) are independent events, then
\[
P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C).
\]

**EXAMPLE 2** Independent Events in a Family

The picture in the margin shows a family that had nine girls in a row. Find the probability of this occurrence.

**Solution** If two or more events are independent, we can find the probability of them all occurring by multiplying their probabilities. The probability of a baby girl is \( \frac{1}{2} \), so the probability of nine girls in a row is \( \frac{1}{2} \) used as a factor nine times.

\[
P(\text{nine girls is a row}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \left( \frac{1}{2} \right)^9 = \frac{1}{512}
\]

The probability of a run of nine girls in a row is \( \frac{1}{512} \). (If another child is born into the family, this event is independent of the other nine and the probability of a girl is still \( \frac{1}{2} \)).

**EXERCISE 2** Find the probability of a family having four boys in a row.
Now let us return to the hurricane problem that opened this section. The Saffir/Simpson scale assigns numbers 1 through 5 to measure the disaster potential of a hurricane’s winds. Table 11.6 describes the scale. According to the National Hurricane Center, the probability that South Florida will be hit by a hurricane or higher in any single year is $\frac{5}{19}$, or approximately 0.26. In Example 3, we explore the risks of living in “Hurricane Alley.”

**EXAMPLE 3** Hurricanes and Probabilities

If the probability that South Florida will be hit by a hurricane in any single year is $\frac{5}{19}$.

a. What is the probability that South Florida will be hit by a hurricane in three consecutive years?

b. What is the probability that South Florida will not be hit by a hurricane in the next ten years?

**Solution**

a. The probability that South Florida will be hit by a hurricane in three consecutive years is

$$P(\text{hurricane and hurricane}) = P(\text{hurricane}) \cdot P(\text{hurricane}) \cdot P(\text{hurricane}) = \frac{5}{19} \cdot \frac{5}{19} \cdot \frac{5}{19} = \frac{125}{6859} \approx 0.018.$$  

b. We will first find the probability that South Florida will not be hit by a hurricane in any single year.

$$P(\text{no hurricane}) = 1 - P(\text{hurricane}) = 1 - \frac{5}{19} = \frac{14}{19} \approx 0.737$$

The probability of not being hit by a hurricane in a single year is $\frac{14}{19}$. Therefore, the probability of not being hit by a hurricane ten years in a row is $\left(\frac{14}{19}\right)^{10}$ used as a factor ten times.

$$P(\text{no hurricanes for ten years}) = P(\text{no hurricane for year 1}) \cdot P(\text{no hurricane for year 2}) \cdot P(\text{no hurricane for year 3}) \cdots P(\text{no hurricane for year 10})$$

$$= \frac{14}{19} \cdot \frac{14}{19} \cdot \frac{14}{19} \cdots \frac{14}{19}$$

$$= \left(\frac{14}{19}\right)^{10} \approx (0.737)^{10} \approx 0.047$$

The probability that South Florida will not be hit by a hurricane in the next ten years is approximately 0.047.

Now we are ready to answer your question:

What is the probability that South Florida will be hit by a hurricane at least once in the next ten years?

Because $P(\text{not } E) = 1 - P(E)$,

$$P(\text{no hurricane for ten years}) = 1 - P(\text{at least one hurricane in ten years}).$$

Equivalently,

$$P(\text{at least one hurricane in ten years}) = 1 - P(\text{no hurricane for ten years})$$

$$= 1 - 0.047 = 0.953.$$  

With a probability of 0.953, it is nearly certain that South Florida will be hit by a hurricane at least once in the next ten years.
THE PROBABILITY OF AN EVENT HAPPENING AT LEAST ONCE

\[ P(\text{event happening at least once}) = 1 - P(\text{event does not happen}) \]

3. If the probability that South Florida will be hit by a hurricane in any single year is \( \frac{1}{10} \),
   a. What is the probability that South Florida will be hit by a hurricane in four consecutive years?
   b. What is the probability that South Florida will not be hit by a hurricane in the next four years?
   c. What is the probability that South Florida will be hit by a hurricane at least once in the next four years?

Express all probabilities as fractions and as decimals rounded to three places.

And Probabilities with Dependent Events

Chocolate lovers, please help yourselves! There are 20 mouth-watering tidbits to select from. What’s that? You want 2? And you prefer chocolate-covered cherries? The problem is that there are only 5 chocolate-covered cherries and it’s impossible to tell what is inside each piece. They’re all shaped exactly alike. At any rate, reach in, select a piece, enjoy, choose another piece, eat, and be well. There is nothing like savoring a good piece of chocolate in the midst of all this chit-chat about probability and hurricanes.

Another question? You want to know what your chances are of selecting 2 chocolate-covered cherries? Well, let’s see. Five of the 20 pieces are chocolate-covered cherries, so the probability of getting one of them on your first selection is \( \frac{5}{20} \), or \( \frac{1}{4} \). Now, suppose that you did choose a chocolate-covered cherry on your first pick. Eat it slowly; there’s no guarantee that you’ll select your favorite on the second selection. There are now only 19 pieces of chocolate left. Only 4 are chocolate-covered cherries. The probability of getting a chocolate-covered cherry on your second try is 4 out of 19, or \( \frac{4}{19} \). This is a different probability than the \( \frac{1}{4} \) probability on your first selection. Selecting a chocolate-covered cherry the first time changes what is in the candy box. The probability of what you select the second time is affected by the outcome of the first event. For this reason, we say that these are dependent events.

Dependent Events

Two events are dependent events if the occurrence of one of them has an effect on the probability of the other.

The probability of selecting two chocolate-covered cherries in a row can be found by multiplying the \( \frac{1}{4} \) probability on the first selection by the \( \frac{4}{19} \) probability on the second selection:

\[ P(\text{chocolate-covered cherry and chocolate-covered cherry}) = P(\text{chocolate-covered cherry}) \cdot P(\text{given that one was selected}) \]

\[ = \frac{1}{4} \cdot \frac{4}{19} = \frac{1}{19} \]

The probability of selecting two chocolate-covered cherries in a row is \( \frac{1}{19} \). This is a special case of finding the probability that each of two dependent events occurs.

And Probabilities with Dependent Events

If \( A \) and \( B \) are dependent events, then

\[ P(A \text{ and } B) = P(A) \cdot P(B \text{ given that } A \text{ has occurred}) \]
EXAMPLE 4 \textbf{An And Probability with Dependent Events}

Good news: You won a free trip to Madrid and can take two people with you, all expenses paid. Bad news: Ten of your cousins have appeared out of nowhere and are begging you to take them. You write each cousin's name on a card, place the cards in a hat, and select one name. Then you select a second name without replacing the first card. If three of your ten cousins speak Spanish, find the probability of selecting two Spanish-speaking cousins.

**Solution** Because \( P(A \text{ and } B) = P(A) \cdot P(B \text{ given that } A \text{ has occurred}) \), then

\[
P(\text{two Spanish-speaking cousins}) = P(\text{speaks Spanish and speaks Spanish})
\]

\[
= P(\text{speaks Spanish}) \cdot P\left(\text{speaks Spanish given that a Spanish-speaking cousin was selected first}\right)
\]

\[
= \frac{3}{10} \cdot \frac{2}{9}
\]

There are ten cousins, three of whom speak Spanish.

After picking a Spanish-speaking cousin, there are nine cousins left, two of whom speak Spanish.

\[
= \frac{6}{90} = \frac{1}{15} \approx 0.067.
\]

The probability of selecting two Spanish-speaking cousins is \( \frac{1}{15} \).

EXAMPLE 4 You are dealt two cards from a 52-card deck. Find the probability of getting two kings.

The multiplication rule for dependent events can be extended to cover three or more dependent events. For example, in the case of three such events,

\[
P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B \text{ given that } A \text{ occurred}) \cdot P(C \text{ given that } A \text{ and } B \text{ occurred}).
\]

EXAMPLE 5 \textbf{An And Probability with Three Dependent Events}

Three people are randomly selected, one person at a time, from five freshmen, two sophomores, and four juniors. Find the probability that the first two people selected are freshmen and the third is a junior.

**Solution**

\[
P(\text{first two are freshmen and the third is a junior}) = P(\text{freshman}) \cdot P\left(\text{freshman given that a freshman was selected first}\right) \cdot P\left(\text{junior given that a freshman was selected first and a freshman was selected second}\right)
\]

\[
= \frac{5}{11} \cdot \frac{4}{10} \cdot \frac{4}{9}
\]

There are 11 people, five of whom are freshmen.

After picking a freshman, there are 10 people left, four of whom are freshmen.

After the first two selections, 9 people are left, four of whom are juniors.

\[
= \frac{8}{99}
\]

The probability that the first two people selected are freshmen and the third is a junior is \( \frac{8}{99} \).
You are dealt three cards from a 52-card deck. Find the probability of getting three hearts.

**Conditional Probability**

We have seen that for any two dependent events $A$ and $B$,

$$P(A \text{ and } B) = P(A) \cdot P(B \text{ given that } A \text{ occurs}).$$

The probability of $B$ given that $A$ occurs is called *conditional probability*, denoted by $P(B \mid A)$.

**COINCIDENCES**

The phone rings and it’s the friend you were just thinking of. You’re driving down the road and a song you were humming in your head comes on the radio. Although these coincidences seem strange, perhaps even mystical, they’re not. Coincidences are bound to happen. Ours is a world in which there are a great many potential coincidences, each with a low probability of occurring. When these surprising coincidences happen, we are amazed and remember them. However, we pay little attention to the countless number of non-coincidences: How often do you think of your friend and she doesn’t call, or how often does she call when you’re not thinking about her? By noticing the hits and ignoring the misses, we incorrectly perceive that there is a relationship between the occurrence of two independent events.

Another problem is that we often underestimate the probabilities of coincidences in certain situations, acting with more surprise than we should when they occur. For example, in a group of only 23 people, the probability that two individuals share a birthday (same month and day) is greater than $\frac{1}{2}$. Above 50 people, the probability of any two people sharing a birthday approaches certainty. You can verify the probabilities behind the coincidence of shared birthdays in relatively small groups by working Exercise 87 in Exercise Set 11.7.

**EXAMPLE 6** Finding Conditional Probability

A letter is randomly selected from the letters of the English alphabet. Find the probability of selecting a vowel, given that the outcome is a letter that precedes h.

**Solution** We are looking for

$$P(\text{vowel} \mid \text{letter precedes h}).$$

This is the probability of a vowel if the sample space is restricted to the set of letters that precede h. Thus, the sample space is given by

$$S = \{a, b, c, d, e, f, g\}.$$  

There are seven possible outcomes in the sample space. We can select a vowel from this set in one of two ways: a or e. Therefore, the probability of selecting a vowel, given that the outcome is a letter that precedes h, is $\frac{2}{7}$.

$$P(\text{vowel} \mid \text{letter precedes h}) = \frac{2}{7}.$$

**EXAMPLE 7** Finding Conditional Probability

You are dealt one card from a 52-card deck.

a. Find the probability of getting a heart, given that the card you were dealt is a red card.
b. Find the probability of getting a red card, given that the card you were dealt is a heart.
Solution

a. We begin with

\[ P(\text{heart} \mid \text{red card}) \]  

The sample space is shown in Figure 11.12. There are 26 outcomes in the sample space. We can get a heart from this set in 13 ways. Thus,

\[ P(\text{heart} \mid \text{red card}) = \frac{13}{26} = \frac{1}{2} \]

b. We now find

\[ P(\text{red card} \mid \text{heart}) \]  

The sample space is shown in Figure 11.13. There are 13 outcomes in the sample space. All of the outcomes are red. We can get a red card from this set in 13 ways. Thus,

\[ P(\text{red card} \mid \text{heart}) = \frac{13}{13} = 1. \]

Example 7 illustrates that \( P(\text{heart} \mid \text{red card}) \) is not equal to \( P(\text{red card} \mid \text{heart}) \). In general, \( P(B \mid A) \neq P(A \mid B) \).

7 You are dealt one card from a 52-card deck.

a. Find the probability of getting a black card, given the card you were dealt is a spade.

b. Find the probability of getting a spade, given the card you were dealt is a black card.

Example 8  Conditional Probabilities with Real-World Data

When women turn 40, their gynecologists typically remind them that it is time to undergo mammography screening for breast cancer. The data in Table 11.7 are based on 100,000 U.S. women, ages 40 to 50, who participated in mammography screening.

| Table 11.7 Mammography Screening on 100,000 U.S. Women, Ages 40 to 50 |
|-----------------|----------------|----------------|
|                  | Breast Cancer | No Breast Cancer | Total  |
| Positive Mammogram |       720     |            6944    |       7664 |
| Negative Mammogram  |        80     |             92,256 |      92,336 |
| Total               |       800     |            99,200   |   100,000 |


Assuming that these numbers are representative of all U.S. women ages 40 to 50, find the probability that a woman in this age range

a. has a positive mammogram, given that she does not have breast cancer.

b. does not have breast cancer, given that she has a positive mammogram.
Solution

a. We begin with the probability that a U.S. woman aged 40 to 50 has a positive mammogram, given that she does not have breast cancer:

\[ P(\text{positive mammogram}|\text{no breast cancer}) \]

This is the probability of a positive mammogram if the data are restricted to women without breast cancer:

<table>
<thead>
<tr>
<th></th>
<th>No Breast Cancer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive Mammogram</td>
<td>6944</td>
</tr>
<tr>
<td>Negative Mammogram</td>
<td>92,256</td>
</tr>
<tr>
<td>Total</td>
<td>99,200</td>
</tr>
</tbody>
</table>

Within the restricted data, there are 6944 women with positive mammograms and 6944 + 92,256, or 99,200 women without breast cancer. Thus,

\[ P(\text{positive mammogram}|\text{no breast cancer}) = \frac{6944}{99,200} = 0.07. \]

Among women without breast cancer, the probability of a positive mammogram is 0.07.

b. Now, we find the probability that a U.S. woman aged 40 to 50 does not have breast cancer, given that she has a positive mammogram:

\[ P(\text{no breast cancer}|\text{positive mammogram}) \]

This is the probability of not having breast cancer if the data are restricted to women with positive mammograms:

<table>
<thead>
<tr>
<th></th>
<th>Breast Cancer</th>
<th>No Breast Cancer</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive Mammogram</td>
<td>720</td>
<td>6944</td>
<td>7664</td>
</tr>
</tbody>
</table>

Within the restricted data, there are 6944 women without breast cancer and 720 + 6944, or 7664 women with positive mammograms. Thus,

\[ P(\text{no breast cancer}|\text{positive mammogram}) = \frac{6944}{7664} \approx 0.906. \]

Among women with positive mammograms, the probability of not having breast cancer is \( \frac{6944}{7664} \), or approximately 0.906.

Use the data in Table 11.7 at the bottom of the previous page to find the probability that a U.S. woman aged 40 to 50

a. has a positive mammogram, given that she has breast cancer.

b. has breast cancer, given that she has a positive mammogram.

Express probabilities as decimals and, if necessary, round to three decimal places.

We have seen that \( P(B|A) \) is the probability that event \( B \) occurs if the sample space is restricted to event \( A \). Thus,

\[ P(B|A) = \frac{\text{number of outcomes of } B \text{ that are in the restricted sample space } A}{\text{number of outcomes in the restricted sample space } A}. \]

This can be stated in terms of the following formula:

**A FORMULA FOR CONDITIONAL PROBABILITY**

\[ P(B|A) = \frac{n(B \cap A)}{n(A)} = \frac{\text{number of outcomes common to } B \text{ and } A}{\text{number of outcomes in } A}. \]
Exercise Set 11.7

Practice and Application Exercises

Exercises 1–26 involve probabilities with independent events.

Use the spinner shown to solve Exercises 1–10. It is equally probable that the pointer will land on any one of the six regions. If the pointer lands on a borderline, spin again. If the pointer is spun twice, find the probability it will land on:

1. green and then red.
2. yellow and then green.
3. yellow and then yellow.
4. red and then red.
5. a color other than red each time.
6. a color other than green each time.

If the pointer is spun three times, find the probability it will land on:
7. green and then red and then yellow.
8. red and then red and then green.
9. red every time.
10. green every time.

In Exercises 11–14, a single die is rolled twice. Find the probability of rolling:
11. a 2 the first time and a 3 the second time.
12. a 5 the first time and a 1 the second time.
13. an even number the first time and a number greater than 2 the second time.
14. an odd number the first time and a number less than 3 the second time.

In Exercises 15–20, you draw one card from a 52-card deck. Then the card is replaced in the deck, the deck is shuffled, and you draw again. Find the probability of drawing:
15. a picture card the first time and a heart the second time.
16. a jack the first time and a club the second time.
17. a king each time.
18. a 3 each time.
19. a red card each time.
20. a black card each time.

21. If you toss a fair coin six times, what is the probability of getting all heads?
22. If you toss a fair coin seven times, what is the probability of getting all tails?

In Exercises 23–24, a coin is tossed and a die is rolled. Find the probability of getting:
23. a head and a number greater than 4.
24. a tail and a number less than 5.
25. The probability that South Florida will be hit by a major hurricane (category 4 or 5) in any single year is 1/60. (Source: National Hurricane Center)
   a. What is the probability that South Florida will be hit by a major hurricane two years in a row?
   b. What is the probability that South Florida will be hit by a major hurricane in three consecutive years?
   c. What is the probability that South Florida will not be hit by a major hurricane in the next ten years?
   d. What is the probability that South Florida will be hit by a major hurricane at least once in the next ten years?
26. The probability that a region prone to flooding will flood in any single year is 1/10.
   a. What is the probability of a flood two years in a row?
   b. What is the probability of flooding in three consecutive years?
   c. What is the probability of no flooding for ten consecutive years?
   d. What is the probability of flooding at least once in the next ten years?

The graph shows that U.S. adults dependent on tobacco have a greater probability of suffering from some ailments than the general adult population. When making two or more selections from populations with large numbers, such as the U.S. adult population or the population dependent on tobacco, we assume that each selection is independent of every other selection. In Exercises 27–32, assume that the selections are independent events.

**Probability That U.S. Adults Suffer From Various Ailments**

<table>
<thead>
<tr>
<th>Ailment</th>
<th>Tobacco-Dependent Population</th>
<th>General Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depression</td>
<td>0.12</td>
<td>0.28</td>
</tr>
<tr>
<td>Frequent Hangovers</td>
<td>0.10</td>
<td>0.20</td>
</tr>
<tr>
<td>Anxiety/Panic Disorder</td>
<td>0.08</td>
<td>0.19</td>
</tr>
<tr>
<td>Severe Pain</td>
<td>0.07</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Source: MARS 2005 OTC/DTC

27. If two adults are randomly selected from the general population, what is the probability that they both suffer from depression?
28. It two adults are randomly selected from the population of cigarette smokers, what is the probability that they both suffer from depression?

29. If three adults are randomly selected from the population of cigarette smokers, what is the probability that they all suffer from frequent hangovers?

30. If three adults are randomly selected from the general population, what is the probability that they all suffer from frequent hangovers?

31. If three adults are randomly selected from the population of cigarette smokers, what is the probability, expressed as a decimal correct to four places, that at least one person suffers from anxiety/panic disorder?

32. If three adults are randomly selected from the population of cigarette smokers, what is the probability, expressed as a decimal correct to four places, that at least one person suffers from severe pain?

Exercises 33–48 involve probabilities with dependent events.

In Exercises 33–36, we return to our box of chocolates. There are 30 chocolates in the box, all identically shaped. Five are filled with coconut, 10 with caramel, and 15 are solid chocolate. You randomly select one piece, eat it, and then select a second piece. Find the probability of selecting

33. two solid chocolates in a row.
34. two caramel-filled chocolates in a row.
35. a coconut-filled chocolate followed by a caramel-filled chocolate.
36. a coconut-filled chocolate followed by a solid chocolate.

In Exercises 37–42, consider a political discussion group consisting of 5 Democrats, 6 Republicans, and 4 Independents. Suppose that two group members are randomly selected, in succession, to attend a political convention. Find the probability of selecting

37. two Democrats.
38. two Republicans.
39. an Independent and then a Republican.
40. an Independent and then a Democrat.
41. no Independents.
42. no Democrats.

In Exercises 43–48, an ice chest contains six cans of apple juice, eight cans of grape juice, four cans of orange juice, and two cans of mango juice. Suppose that you reach into the container and randomly select three cans in succession. Find the probability of selecting

43. three cans of apple juice.
44. three cans of grape juice.
45. a can of grape juice, then a can of orange juice, then a can of mango juice.
46. a can of apple juice, then a can of grape juice, then a can of orange juice.

47. no grape juice.
48. no apple juice.

In Exercises 49–56, the numbered disks shown are placed in a box and one disk is selected at random.

Find the probability of selecting

49. a 3, given that a red disk is selected.
50. a 7, given that a yellow disk is selected.
51. an even number, given that a yellow disk is selected.
52. an odd number, given that a red disk is selected.
53. a red disk, given that an odd number is selected.
54. a yellow disk, given that an odd number is selected.
55. a red disk, given that the number selected is at least 5.
56. a yellow disk, given that the number selected is at most 3.

The table shows the outcome of car accidents in Florida for a recent year by whether or not the driver wore a seat belt. Use the data to solve Exercises 57–60. Express probabilities as fractions and as decimals rounded to three places.

<table>
<thead>
<tr>
<th></th>
<th>Wore Seat Belt</th>
<th>No Seat Belt</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driver Survived</td>
<td>412,768</td>
<td>162,527</td>
<td>574,895</td>
</tr>
<tr>
<td>Driver Died</td>
<td>510</td>
<td>1601</td>
<td>2111</td>
</tr>
<tr>
<td>Total</td>
<td>412,878</td>
<td>164,128</td>
<td>577,006</td>
</tr>
</tbody>
</table>


57. Find the probability of surviving a car accident, given that the driver wore a seat belt.
58. Find the probability of not surviving a car accident, given that the driver did not wear a seat belt.
59. Find the probability of wearing a seat belt, given that a driver survived a car accident.
60. Find the probability of not wearing a seat belt, given that a driver did not survive a car accident.
In Exercises 61–72, we return to the table showing the distribution, by marital status and gender, of the 235.8 million Americans ages 18 or older.

**Marital Status of the U.S. Population, Ages 18 or Older, in Millions**

<table>
<thead>
<tr>
<th></th>
<th>Never Married</th>
<th>Married</th>
<th>Widowed</th>
<th>Divorced</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>37.5</td>
<td>64.7</td>
<td>2.7</td>
<td>9.6</td>
<td>114.5</td>
</tr>
<tr>
<td>Female</td>
<td>31.7</td>
<td>65.2</td>
<td>11.2</td>
<td>13.2</td>
<td>121.3</td>
</tr>
<tr>
<td>Total</td>
<td>69.2</td>
<td>129.9</td>
<td>13.9</td>
<td>22.8</td>
<td>235.8</td>
</tr>
</tbody>
</table>

Source: U.S. Census Bureau

81. If a fourth child is born into a family with three boys, the odds in favor of a girl are better than 1:1.

82. In a group of five men and five women, the probability of randomly selecting a man is \( \frac{2}{5} \), so if I select two people from the group, the probability that both are men is \( \frac{2}{5} \cdot \frac{1}{4} \).

83. I found the probability of getting rain at least once in ten days by calculating the probability that none of the days have rain and subtracting this probability from 1.

84. I must have made an error calculating probabilities because \( P(A|B) \) is not the same as \( P(B|A) \).

85. If the probability of being hospitalized during a year is 0.1, find the probability that no one in a family of five will be hospitalized in a year.

86. If a single die is rolled five times, what is the probability it lands on 2 on the first, third, and fourth rolls, but not on either of the other rolls?

87. **Probabilities and Coincidence of Shared Birthdays**
   a. If two people are selected at random, the probability that they do not have the same birthday (day and month) is \( \frac{365}{365} \cdot \frac{364}{365} \). Explain why this is so. (Ignore leap years and assume 365 days in a year.)
   b. If three people are selected at random, find the probability that they all have different birthdays.
   c. If three people are selected at random, find the probability that at least two of them have the same birthday.
   d. If 20 people are selected at random, find the probability that at least 2 of them have the same birthday.
   e. How large a group is needed to give a 0.5 chance of at least two people having the same birthday?

88. Nine cards numbered from 1 through 9 are placed into a box and two cards are selected without replacement. Find the probability that both numbers selected are odd, given that their sum is even.

89. If a single die is rolled twice, find the probability of rolling an odd number and a number greater than 4 in either order.

**Group Exercises**

90. Do you live in an area prone to catastrophes, such as earthquakes, fires, tornados, hurricanes, or floods? If so, research the probability of this catastrophe occurring in a single year. Group members should then use this probability to write and solve a problem similar to Exercise 25 in this exercise set.

91. Group members should use the table for Exercises 61–72 to write and solve four probability problems different than those in the exercises. Two should involve **or** (one with events that are mutually exclusive and one with events that are not), one should involve **and**—that is, events in succession—and one should involve conditional probability.